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# Plasmas in Quasi-Static External Electric Fields

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# PLASMAS IN QUASI-STATIC EXTERNAL ELECTRIC FIELDS

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## ABSTRACT

This work develops some practical approximations needed to simulate a high plasma density volume bounded by walls made of dielectrics or metals which may be either biased or floating in potential. Solving Poisson's equation in both the high-density bulk and the sheath region poses a difficult computational problem due to the large electron plasma frequency. A common approximation is to assume the electric field is computed in the ambipolar approximation in the bulk and to couple this to a sheath model at the boundaries. Unfortunately, this treatment is not appropriate when some surfaces are biased with respect to others and a net current is present within the plasma. This report develops some ideas on the application of quasi-static external electric fields to plasmas and the self-consistent treatment of boundary conditions at the surfaces. These constitute a generalization of Ohm's law for a plasma body that entails solving for the internal fields within the plasma and the potential drop and currents through the sheaths surrounding the plasma.

## I. INTRODUCTION AND MOTIVATION

The motivation for this work originates in the desire to simulate a high plasma density region which is bounded by thin sheaths near walls made of dielectrics or metals which may be either potential biased or floating. It is well known that solving Poisson's equation in both the high-density bulk and the sheath region poses a difficult computational problem. A common approximation (in semiconductor processing plasmas, for example) is to assume the electric field is computed in the ambipolar approximation (zero net current) in the bulk and couple this to a sheath model at the boundaries. Unfortunately, this treatment is not appropriate when some surfaces are biased with respect to others and a net current is present in the plasma. This report develops some ideas on the application of static external electric fields to plasmas and the self-consistent treatment of boundary conditions at the surfaces. Together these constitute a generalization of Ohm's law for a plasma body that entails solving for the internal fields within the plasma together with the potential drop and currents through the sheaths surrounding the plasma. This report targets an audience which includes those wanting a concise summary of the treatment of quasi-static electric fields in dense plasmas.

## II. BASIC PLASMA EQUATIONS

While in principle the analysis described here can be cast in terms of a full kinetic treatment, all the theory here is for a fluid description of plasma evolution [1] in a background gas with the electrons treated in the drift-diffusion approximation. The drift diffusion approximation for the electrons makes the problem considerably less stiff, since the removal of electron inertia eliminates the inverse plasma frequency as the fastest timescale. Application of the equations presented here to other descriptions (such as kinetic or gas flow) will require some modifications. In particular, it may be necessary to add terms for the flow and density variations of the background neutral gas and the spatial variation of the electron temperature. If the ionization fraction is high, there are terms representing the interaction of the electrons with the ions which should be included. An estimate of the ionization fraction at which the electrons interact equally with ions and neutrals is $N_i/N_0=0.008\, T_e^2$, with $T_e$ in eV. These omissions will be noted in the text at appropriate places. None of these omissions invalidate the basic method of solution for the fields within the plasma. In order to simulate the dynamic problem, the displacement current would need to be included in the formulation. This dynamical situation would arise in the case of a conductor biased at rf frequencies, as is common in semiconductor processing devices.

The ion continuity equation (iCE) is

$$\dot{n}_i + \vec{\nabla} \cdot (n_i \vec{u}_i) = r_i \tag{1}$$

where $n_i$ is the ion number density, $\vec{u}_i$ is the ion fluid velocity, and $r_i$ is the volume ionization source rate. The electron continuity equation (eCE) is:

$$\dot{n}_e + \vec{\nabla} \cdot (n_e \vec{u}_e) = r_i \qquad (2)$$

where $n_e$ is the electron number density and $\vec{u}_e$ is the electron fluid velocity. The ion momentum equation (iME) is

$$\dot{\vec{u}}_i + (\vec{u}_i \cdot \vec{\nabla})\vec{u}_i = \frac{e}{m_i}\vec{E} - v_i \vec{u}_i \, , \qquad (3)$$

where $\vec{E} = -\vec{\nabla}\phi$ is the electric field, $v_i$ is the sum of momentum transfer and ionization collision frequencies, and the ion diffusion term, which is almost always small, has been neglected. There is a constant background neutral density which is assumed larger than the plasma density and nearly stationary. Corrections can be introduced for these effects. If we neglect electron inertia in the electron momentum equation, we arrive at the electron drift-diffusion equation (eDDE):

$$\vec{u}_e = -\mu_e \vec{E} - D_e \frac{1}{n_e}\vec{\nabla} n_e, \qquad (4)$$

where $\mu_e$ is the electron mobility and $D_e$ is the electron diffusivity. Again this neglects collisions with the ions and the flow of the background neutral gas. Poisson's equation describes the collective interaction of the charged species:

$$-\nabla^2 \phi = \vec{\nabla} \cdot \vec{E} = \frac{e}{\varepsilon_o}(n_i - n_e). \qquad (5)$$

We are assuming that the plasma is dominated by a single positive ion species. The particle electric current density, $\vec{J}$, within the plasma is given by the sum of the ion and electron currents:

$$\vec{J} = e(n_i \vec{u}_i - n_e \vec{u}_e) \equiv \vec{j}_i + \vec{j}_e. \qquad (6)$$

All other symbols are as usually defined.

Eqs.(1)-(6) can be solved numerically as written. This solution would include the plasma bulk and the sheath regions, with boundary conditions (b.c.) to be imposed at the actual physical walls or electrodes in contact with the plasma. The displacement current and electron inertia might be needed in cases with high-frequency fields applied to the plasma - such situations will not be considered here. In general for higher density plasmas with slowly varying applied fields, the plasma is quasi-neutral except in the thin sheaths. This allows elimination of the Poisson equation with the associated large electron plasma frequency as done in the next sections. If the plasma density is very low, though,

the restoring field due to charge separation is weak and the plasma will not remain quasi-neutral. In such a case the approximations discussed here are not applicable.

## II.A QUASI-NEUTRAL APPROXIMATION (QNA)

With some care as to all the consequences, we can just set $n_e = n_i$ in the above equations to derive this approximation. The QNA should be derived by an ordering approximation so that the size and importance of various terms can be assessed. Eq.(1) is unchanged, but Eq.(2) becomes:

$$\dot{n}_i + \vec{\nabla} \cdot (n_i \vec{u}_e) = r_i \tag{7}$$

which is subtracted from Eq.(1) to give

$$\vec{\nabla} \cdot (n_i (\vec{u}_i - \vec{u}_e)) = 0 \tag{8}$$

or,

$$\vec{\nabla} \cdot \vec{J} = 0 \, , \tag{9}$$

where,

$$\vec{J} = e\, n_i\, (\vec{u}_i - \vec{u}_e), \tag{10}$$

using the definition of $\vec{J}$ from Eq.(6) in the quasi-neutral limit. The current in the plasma obeys Eq.(9), which can be reformulated as a total surface integral using the divergence theorem:

$$0 = \int_V d^3r\, \vec{\nabla} \cdot \vec{J} = \int_S d^2s\, \hat{n} \cdot \vec{J} = \sum_{S_n} dA_n\, j_n \tag{11}$$

expressing the total current conservation throughout the volume in terms of the currents on each surface element labeled by $n$. These $j_n$ are obtained from boundary conditions and are functions of the local plasma density, plasma potential, and b.c. potentials. The iME in Eq.(3) is unchanged by quasi-neutrality. The eDDE in Eq.(4) now contains the ion density in the diffusion term:

$$\vec{u}_e = -\mu_e \vec{E} - D_e \frac{1}{n_i} \vec{\nabla} n_i, \tag{12}$$

This electron fluid velocity is then used in the relations (9) and (10) above. An important point is that the bulk plasma current can be specified throughout the

plasma in the QNA with the only restriction made by the source-free condition in Eq.(9).

What does this mean? In the basic equations of motion, Eqs.(1)-(6), the Poisson equation determines the field and potential from the charge density. Thus there is self-consistency between the field due to the space charge and the acceleration of the electrons and ions. Now in the QNA there is limited self-consistency. One can *specify* an arbitrary $\vec{J}$, subject to source-free conditions in Eq.(9) and the yet-to-be-discussed b.c. on the QNA, and solve for the ion and electron motion *without any feedback* to the imposed current. This would involve integration with Eqs.(1), (3), (10), and (12), where $\vec{u}_e$ is determined from $\vec{u}_i$ and $\vec{J}$ by Eq.(10), and $\vec{E}$ from $\vec{u}_e$ and $n_i$ in Eq.(12). The field would then enter the iME to drive the ion solution.

## III. ANALYSIS OF THE QNA

The computational domain for the QNA must exclude the transition region between the Bohm point and the physical wall. Since some physical quantities vary strongly across this region, we must accordingly revise the b.c. to be applied to the QNA solution. The QNA contains both the ambipolar and strong-internal field limits, both of which are described in the Appendix. We discuss it from a more practical point of view here, including the imposition of sheath and surface b.c. Combining Eqs.(10) and (12) to eliminate $\vec{u}_e$:

$$\vec{J} = e n_i \vec{u}_i + e n_i \mu_e \vec{E} + e D_e \vec{\nabla} n_i , \tag{13}$$

or, in terms of $\vec{E}$,

$$\vec{E} = \frac{\vec{J}}{e n_i \mu_e} - \frac{1}{\mu_e} \vec{u}_i - \frac{kT_e}{e} \frac{1}{n_i} \vec{\nabla} n_i . \tag{14}$$

The Einstein identity, $D_e / \mu_e = kT_e / e$, was used in Eq.(14). When combined with the iCE and the iME in Eqs.(1) and (3), this furnishes a *complete* description of the plasma response when the current density is a prescribed quantity. Of course the current density in any real situation is an unknown which is dependent on the potential differences applied to the plasma. That is why the QNA must be connected across the sheaths to the walls where the potentials are defined. This will be done next.

## III.A. BOUNDARY CONDITIONS

Boundary conditions can be formulated quite generally in terms of Eqs.(1)-(6), but these equations will not be solved as they stand due to severe numerical problems related to the plasma frequency (or dielectric response frequency) and the thin sheath region. Here we will be concerned with imposing sheath-approximation boundary conditions (b.c.) to the bulk plasma when it is described

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by the QNA. The QNA is not valid within the sheath regions. We will use a label of $s$ to denote a "surface" region for quantities evaluated at the Bohm point and a label of $w$ to denote "wall" for quantities which are evaluated at the actual physical surface when they are different across the sheath. Typically, only the potential and electron density change abruptly across the sheath.

The simplest of the situations can be described as follows. Let the plasma be contained within a volume defined by surfaces with specified potentials on some elements (conducting surfaces) and unknown voltages on others (dielectrics or floating conductors). Currents are not normally b.c. quantities, but are determined in response to the imposed voltages. However there may be zero-current conditions imposed on the dielectric surfaces or insulated metal surfaces. Some of the boundaries may not be actual surfaces, but regions where the plasma flux and density are specified.

From analysis of the plasma sheath, one knows that the following is approximately true for any surface embedded within the plasma. First of all, the ion current at the Bohm point is given by

$$j_{s,i} = e\,n_i\,u_B \tag{15}$$

at a particular boundary point in terms of the neighboring ion density and Bohm velocity [2] in the plasma. This ion current is nearly constant across the sheath. The Bohm velocity is a function of the local electron temperature and ion mass: $u_B = \sqrt{kT_e / m_i}$ . The velocity is directed toward the surface. The electron current is given in terms of the potential drop from the plasma bulk, defined at the Bohm point, to the physical surface. The Bohm point lies near the physical surface in contact with the plasma, and it is the transition point between the quasi-neutral bulk and the non-neutral, charge-separated sheath region. The potential of the surface may be known or unknown. This is represented as

$$j_{s,e} = -\tfrac{1}{4}e\,n_i\,u_{Te}\exp(-e\Delta V / kT_e), \tag{16}$$

where $\Delta V$ is taken to be a positive quantity for a wall potential $V_w$ less than the nearby plasma potential. Thus $\Delta V = \phi_s - V_w$. $\phi_s$ denotes the value of the plasma potential at the Bohm point prior to the sheath drop. The current through the surface element is the sum of the ion and electron current. $u_{Te}$ is the thermal electron velocity, $u_{Te} = \sqrt{8kT_e / \pi m_e}$ . Displacement current can be neglected in this study as we are not dealing with rapidly varying applied potentials.

All dielectric surfaces and their free-space analogs are required to have zero current conditions because they rapidly charge up to such a voltage that the current is zero. Then the potential drops at those surface elements are evaluated in terms of the local ion density and electron temperature by combining Eqs.(15) and (16) to give zero current. The voltage-specified, metal-like surface elements will have a non-zero current determined by the potential drop from the nearby plasma to the element. It is now obvious that we *must* solve for the potential field

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throughout the plasma bulk (but not in the sheaths) because the currents at the metal surfaces are expressed in terms of the potential drop between the plasma potential and the potential specified in the b.c. Designate the potential field as $\phi$. It solves:

$$\vec{\nabla}\phi = -\vec{E}, \tag{17}$$

where $\vec{E}$ is given by the QNA approximation in Eq.(14). The result is the same as using Eq.(17) to replace $\vec{E}$ on the RHS of Eq.(13). What we now have for the "electronic" part of the problem are Eqs.(9), (13), and (17), with current b.c. expressed in terms of the potential as may be expressed using Eqs.(15) and (16).

### III.B. SOLVING THE EQUATIONS

The question is how to solve this combination of equations without undo difficulty. The solution is nonlinear because of the exponential dependence of the surface currents on the unknown plasma potential in the b.c. The equations for the potential and the current can be collected as the set:

$$\vec{J} = \vec{J}_{i+ed} - e\mu_e n_i \vec{\nabla}\phi,$$

$$\vec{\nabla} \cdot \vec{J} = 0, \tag{18}$$

$$\vec{J} \cdot \hat{n}_s \bigg|_{surface} = e n_i (u_B - \tfrac{1}{4} u_{Te} \exp(-e\Delta V / kT_e)),$$

where

$$\vec{J}_{i+ed} \equiv e n_i \vec{u}_i + e D_e \vec{\nabla} n_i \tag{19}$$

is the total ion and electron diffusion current, which may be regarded as known from the "ion part" of the plasma solution. One may combine the first and second members of Eq.(18) to form the Poisson-like equation for the potential:

$$\vec{\nabla} \cdot (n_i \vec{\nabla}\phi) = \frac{1}{e\mu_e} \vec{\nabla} \cdot \vec{J}_{i+ed}. \tag{20}$$

This equation is "complete" as it stands. Of course the b.c. must be incorporated by using the third member of Eq.(18). Eq.(20) is just a restatement of Eq.(18) except for the b.c. Because the equations are linear in $\phi$, except for the b.c., we can superimpose a particular solution, $\phi_p$, and a homogeneous solution, $\phi_h$. Substitution of the superposition into the first member of Eq.(18) shows that, if the particular solution solves:

$$e\,\mu_e\,n_i\,\vec{\nabla}\phi_p = \vec{J}_{i+ed},\tag{21}$$

then the homogeneous solution must solve:

$$e\,\mu_e\,n_i\,\vec{\nabla}\phi_h = -\vec{J}\,,$$
$$\vec{\nabla}\cdot(n_i\,\vec{\nabla}\phi_h) = 0\,.\tag{22}$$

Eq.(21) insures that Eq.(20) is satisfied, of course. Moreover, Eq.(21) uniquely defines $\phi_p$ up to an additive constant. Eq.(22) shows that $\phi_h$ is a homogeneous solution of Eq.(20), *i.e.* with the RHS set to zero. The first member of Eq.(22) cannot be solved directly since $\vec{J}$ is an unknown within the volume. From this particular solution and homogeneous solution we can match the b.c. All b.c. are in terms of $\vec{J}$ which appears in the equation for the homogeneous solution, but the nonlinear dependence on potential requires both the homogeneous and particular solutions.

### III.C. PLASMA WITHOUT APPLIED FIELD

Consider a special case of the plasma problem where all boundaries are dielectrics or free space where the current b.c. is zero current everywhere. Then $\vec{J} = 0$ and Eq.(22) allows us to set $\phi_h$ to zero or a constant. There is no need to find $\phi_p$ as the electric field is all that is needed for the ion EOM, and it may be solved directly from Eq.(21):

$$\vec{E} = -\vec{\nabla}\phi_p = -\frac{1}{e\mu_e n_i}\vec{J}_{i+ed}$$
$$= -\frac{1}{\mu_e}\vec{u}_i - \frac{kT_e}{e\,n_i}\vec{\nabla}n_i \approx -\frac{kT_e}{e\,n_i}\vec{\nabla}n_i\,.\tag{23}$$

This is the ambipolar diffusion result for the internal field. An alternative terminology for (23) is the Langmuir-Tonks equation for the electric field.

### III.D. PLASMA WITH WEAK APPLIED FIELD

The equations for the field can be linearized if we assume that the applied field is weak. Exactly how "weak is weak" is to be determined. Note that if iterative methods are employed (i.e. fully-implicit Newton-like methods) the linearization step is not necessary, indeed the weakness of the field becomes irrelevant, and will have greater flexibility in the range of applied fields we can consider. Consider the total ion and electron current through a surface region *s*, obtained by combining Eqs.(15) and (16) and shown in the last member of Eq.(18). This is:

$$\vec{J} \cdot \hat{n}_s \bigg|_{surface} = e n_i \left( u_B - \tfrac{1}{4} u_{Te} \exp(-e(\phi_s - V_w) / kT_e) \right), \qquad (24)$$

where $\hat{n}_s$ is an outward normal to the surface. $V_w$ is the value of the applied voltage at the physical surface element labeled $w$ which is next to $s$. We now write $\phi_s$ as a correction to the value of the potential at the surface, $\phi_s^{(0)}$, that gives zero current through the surface element:

$$\phi_s = \phi_s^{(0)} + \phi_s - \phi_s^{(0)},$$

$$\phi_s^{(0)}(V_w) = V_w + \frac{kT_e}{e} \ln\left( \frac{u_{Te}}{4 u_B} \right), \qquad (25)$$

and assume that $\phi_s$ is close to $\phi_s^{(0)}$ in order to justify an expansion of the current b.c. formula:

$$\vec{J} \cdot \hat{n}_s \bigg|_{surface} \approx e n_i u_B \frac{e(\phi_s - \phi_s^{(0)})}{kT_e}, \qquad (26)$$

This linearization is to be applied to all surface elements where the current is nonzero because of voltage b.c. If the current is constrained to be zero as for dielectric boundaries, it is exact.

Eq.(20) may be solved subject to b.c. derived from Eqs.(18), (25), and (26) to give the Cauchy b.c. description of the linearized equations for the field:

$$\vec{\nabla} \cdot (n_i \vec{\nabla} \phi) = \frac{1}{e \mu_e} \vec{\nabla} \cdot \vec{J}_{i+ed} \quad ,$$

$$u_B \frac{e}{kT_e} \phi_s + \mu_e \vec{\nabla} \phi_s \cdot \hat{n}_s = \frac{1}{e n_i} \vec{J}_{i+ed} \cdot \hat{n}_s + u_B \frac{e}{kT_e} \phi_s^{(0)}(V_w) \qquad (27)$$

$$= \vec{u}_i \cdot \hat{n}_s + D_e \frac{1}{n_i} \vec{\nabla} n_i \cdot \hat{n}_s + u_B \frac{e}{kT_e} \phi_s^{(0)}(V_w) \quad .$$

All quantities in the latter expression are evaluated at the surface. Recall that the quantity $\vec{J}_{i+ed}$ is evaluated from the ion part of the total solution procedure. Eq.(27) is the generalized Ohm's law [4] appropriate for the flow of current through a plasma subjected to voltage boundary conditions at the surface. Of course a simple Ohm's law cannot appear until the current in Eq.(18) is connected to the applied voltages that are the b.c. used to solve Eq.(27). We will later give example solutions that make the current-to-voltage relations more transparent.

It is our belief that Eq.(27) constitutes a starting point for numerical solution of the plasma in an external field. Examination of the linearization approximation shows that it should be valid for current densities less than the order of the ion saturation (Bohm) current. This can be seen directly in Eq.(26) where the size of the exponential is related to the current through the surface.

## IV. EXAMPLE SOLUTIONS

An example is sometimes worth many words. The first example solution is for the field within a 1D (one dimensional) plasma subjected to potential b.c. on the walls. Generally we will be solving the equations with linearized b.c. given in Eq.(27). Consider a 1D plasma with constant ion density, stationary ions, and constant electron temperature. This implies, where primes are used to denote spatial derivatives:

$$n_i = \text{constant}, \quad u_i = 0, \quad J_{i+ed} = 0,$$

$$\nabla^2 \phi = \phi'' = 0, \quad \phi(x) = A + Bx, \quad B = -E,$$

(28)

where we have written out the general solution to the Laplace-like Eq.(27). The b.c. are imposed: $V_0$ at $x = x_0$ and $V_1$ at $x = x_1$. The b.c. in Eq.(27) take the form:

$$\phi(x_0) - \frac{kT_e}{e} \frac{\mu_e}{u_B} \phi'(x_0) = \phi_0^{(0)},$$

$$\phi(x_1) + \frac{kT_e}{e} \frac{\mu_e}{u_{B\,1}} \phi'(x_1) = \phi_1^{(0)},$$

(29)

from which one can determine the unknown constants $A$ and $B$ in the Laplace solution. $\phi_s^{(0)}$ is defined in Eq.(25). The difference of the $\phi_s^{(0)}$ is just the difference of the applied voltages because of the constant electron temperature assumption. The potential and field are found to be:

$$\phi(x) = \frac{V_0 + V_1}{2} + V_p - E\left( x - \frac{x_0 + x_1}{2} \right),$$

$$E = -\frac{V_1 - V_0}{x_1 - x_0 + \lambda},$$

(30)

where

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$$\lambda = 2\frac{kT_e}{e}\frac{\mu_e}{u_B} = 2\frac{D_e}{u_B},$$

$$V_p = \frac{kT_e}{e}\ln\left(\frac{u_{Te}}{4u_B}\right).$$

(31)

$V_p$ is the usual potential drop in a sheath. Note that the vacuum field (obtained by setting $\lambda = 0$ in Eq.(30)) is screened by the presence of $\lambda$, but not in a manner that one might expect: the screening in this model example *does not depend on the plasma density* within the system! The effect of the plasma does not go away as $n_i$ becomes small until the sheath boundaries become so thick that they become wide and invalidate the separation of the region into bulk and sheaths. Another interpretation of $\lambda$ from Eq.(31) is that it is twice the characteristic length to produce an electron diffusion velocity equal to the ion Bohm velocity. This is on the order of 100 or more electron mean free paths.

One can use this example to put another condition on the validity of the linearization. The plasma potential should aways exceed the value of the applied voltage on the boundaries; otherwise the currents in Eqs.(15) and (16) in the Bohm sheath are not valid. If we evaluate the potential given in Eq.(30) and require, say, that $\phi(x_1) > V_1$, we find that this requires:

$$\frac{V_1 - V_0}{2V_p} < 1 + \frac{x_1 - x_0}{\lambda}.$$

(32)

For small $\lambda$ this is little concern, but for larger $\lambda$ the voltage difference is restricted to twice the size of the plasma sheath drop. An evaluation of the size of $\lambda$ for a typical $Cl^+$ plasma shows that

$$\frac{x_1 - x_0}{\lambda} \approx 0.003\frac{x_1 - x_0}{\lambda_{MFP}}$$

(33)

where $\lambda_{MFP}$ is the electron mean free path involved in the electron mobility. For a $H^+$ plasma the numerical constant in Eq.(33) is 0.02. Thus for low pressure (mTorr) plasmas of cm dimension, we may expect that the ratio in Eq.(33) is small and that the applied potential difference is required to be of the size of the plasma potential or less in order that the linearization be accurate. The current through this example problem is given by:

$$J = e\mu_e n_i E = -\frac{e\mu_e n_i}{x_1 - x_0 + \lambda}(V_1 - V_0)$$

$$= \frac{e\mu_e n_i}{x_1 - x_0 + \lambda}(x_1 - x_0)E_{applied}$$

(34)

which identifies the conductivity, $\sigma$ , as:

$$\sigma = \frac{e\mu_e n_i}{1 + \lambda / (x_1 - x_0)}.$$

(35)

Again the "simple" case is obtained by setting $\lambda = 0$, whereas the effect of the sheaths lowers the conductivity.

As a second example, consider the free-space boundary or the b.c. at any dielectric interface where the total current is zero. In this case the linearization is exact and the Cauchy b.c. in Eq.(27) reduce to the Neumann b.c. obtained by inserting the zero-current condition, $\phi_s = \phi_s^{(0)}$:

$$\mu_e \vec{\nabla}\phi_s \cdot \hat{n}_s = \frac{1}{en_i}\vec{J}_{i+ed} \cdot \hat{n}_s ,$$

$$\vec{\nabla}\phi_s \cdot \hat{n}_s = \frac{1}{\mu_e}\vec{u}_i \cdot \hat{n}_s + \frac{kT_e}{e}\frac{1}{n_i}\vec{\nabla}n_i \cdot \hat{n}_s ,$$

(36)

$$\approx \frac{kT_e}{e}\frac{1}{n_i}\vec{\nabla}n_i \cdot \hat{n}_s.$$

This is the same form as the ambipolar approximation for the bulk field obtained in Eq.(23). Thus, if only dielectric b.c. are present, we may solve for the ambipolar field directly as done in Section II.A. In the general case of conducting and zero-current boundaries, one must solve Eq.(27) with the appropriate b.c. applied for all boundaries.

Altogether, we have reduced the computation of the plasma in a external field to the solution of the Poisson-like Eq.(27) with either Cauchy or Neumann b.c. with the only significant approximation being the linearization of the plasma current at the sheaths.

## V. DISCUSSION

What is done in this work is based on the QNA for the plasma bulk, the utilization of sheath approximations for the boundaries, and the linearization of the sheath equations themselves. The linearization of the sheath equations is probably not a severe problem since the actual currents drawn through the plasma will be limited to ion saturation currents.

The major new feature of these notes is the necessity to solve the Poisson-like generalized Ohm's law Eq.(27) to determine the electric field within the plasma bulk. Since this is a standard equation for certain types of Poisson solvers, this is not a major obstacle. However it appears that boundary Green's function methods are not applicable.

## VI. APPENDIX

In this Appendix, we consider two limiting forms of the electric field equation which are commonly in use.

### AMBIPOLAR LIMIT

This is an approximation closely related to the QNA which is very useful for bulk plasma simulations when it is valid. The basic assumption is that the electrons evolve due to a balance between the charge-separation field and the spreading due to their kinetic diffusion. From Eq. (12), assume that the mobility and diffusion terms dominate the equation. Then one solves for $\vec{E}$ [3]:

$$\vec{E} = -\frac{kT_e}{e}\frac{1}{n_i}\vec{\nabla}n_i \qquad (A-1)$$

in terms of the ion (plasma) density. This equation can be expressed in terms of a pressure gradient if the more complete fluid equations are used for the iME. One is left with Eqs.(1), (3), and (A-1) to solve. The total current $\vec{J}$ is small because of the assumptions. Arbitrary currents can not be imposed because the fields could exceed the ambipolar field in Eq.(13) and invalidate the assumption.

### STRONG-INTERNAL-FIELD LIMIT

In the case that there is an internal field within the plasma stronger than the field due to the electron diffusion gradients, one can write down a very simple set of equations describing the plasma motion. Consider the conditions on the electron fluid velocity:

$$|\vec{u}_e| >> |\vec{u}_i|$$

$$|\vec{u}_e| >> D_e \frac{1}{n_i}|\vec{\nabla}n_i| \qquad (A-2)$$

such that Eqs.(10) and (12) reduce to

$$\vec{J} \approx \vec{j}_e \approx -e n_i \vec{u}_e \approx e n_i \mu_e \vec{E} \ . \qquad (A-3)$$

This relation says that the current, which is dominated by the electrons, is given by the electron conductivity and the electric field. No other condition determines the field except that it sustains the imposed current. Now one can solve Eqs.(1), (3), and (A-3) with $\vec{J}$, $\vec{E}$, or $\vec{u}_e$ arbitrarily specified, subject to Eq. (9) and b.c. Notice that this has broken the electrons out of the loop of self-consistency completely. The only constraint on $\vec{J}$ is the source-free condition of Eq.(9).

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## ACKNOWLEDGMENTS

## REFERENCES

[1] Braginskii, S.I. "Transport Properties in a Plasma" in M.A. Leontovich (ed.), "Review of Plasma Physics" Consultants Bureau, New York, 1965.

[2] Bohm, D. "Minimum Kinetic Energy for a Stable Sheath" in: The Characteristics of Electrical Discharges in Magnetic Fields, A. Guthrie and R.K. Wakerling eds., McGraw Hill, New York (1949)

[3] L. Tonks, and I. Langmuir, "A General Theory of the Plasma of an Arc", Phys. Rev. **34**: p876 (1929)

[4] Generalized Ohm's laws are frequently used in the mathematical description of discharges. See, for example Borghi, C.A., Cristofolini, A., and Ribani, P.L., "Analysis of magneto-plasma dynamic transients in a combustion gas magnetohydrodynamic generator" Phys. Plasmas, **4**, (8) p 3082 (1997)

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